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ABSTRACT

Twenty-six structural variables were defined and investigated using a set of algebraic word problems solved by 96 college students. The study attempted to identify a small independent well-defined set of arithmetic, linguistic, and algebraic structural variables which account for a maximum amount of the variance of the observed probability correct of algebra word problems. Findings showed that one linguistic variable, two algebraic variables, and three arithmetic variables entered in the first six steps of a stepwise linear regression. Five of the six variables had significant t-values at the .05 level or lower. Six structural variables defined in terms of the number of words in the largest sentence, the logical transitivity of the unknowns, the recall of formulas, the number of digits in quotients, the number of transpositions, and the type of arithmetic operations seem to account for a large amount of the variance (R squared = .80) of the observed probability correct of algebra word problems. (Author/DT)

Abstract

An Analysis of Arithmetic, Linguistic, and Algebraic Structural Variables That Contribute to Problem Solving Difficulty in Algebra Word Problems

by

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Twenty-six structural variables were defined and investigated using a set of algebraic word problems solved by college students. The study attempted to identify a small independent well-defined set of arithmetic, linguistic, and algebraic structural variables which account for a maximum amount of the variance of the observed probability correct of algebra word problems. The study found one linguistic variable, two algebraic variables, and three arithmetic variables entered in the first six steps of a stepwise linear regression. Five of the six variables had significant t-values at the .05 level or lower.

Several structural variables that were found to be robust in studies in the elementary grades with arithmetic word problems were found to be robust in the present study. Six structural variables defined in terms of the number of words in the largest sentence, the logical transitivity of the unknowns, the recall of formulas, the number of digits in quotients, the number of transpositions, and the type of arithmetic operations seem to account for a large amount of the variance ($R^2 = .80$) of the observed probability correct of algebra word problems.

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Several studies have investigated structural and linguistic variables (Suppes, Jerman, and Brian, 1968; Suppes, Loftus, and Jerman, 1969; Loftus, 1970; Jerman, 1971; Jerman and Rees, 1972; Jerman and Mirman, 1972; Jerman, 1972; Krushinski, 1973). The studies using structural variables attempted to account for the variance in the observed probability correct of arithmetic word problems using a stepwise linear regression. An underlying purpose of structural variable studies was to identify a small set (about six) of independent well-defined structural variables that could be used in the generation of word problems of a predictable level of difficulty.

In an arithmetic word problem structural variable study Jerman (1972) suggested that a structural variable study should be conducted "on an entirely different set of problems." In the same study Jerman also proposed that a similar investigation be made in the upper grades.

Hence, the purpose of the present study was to identify a small independent well-defined set of arithmetic, linguistic, and algebraic structural variables which account for the observed probability correct of algebra word problems with college students.

Twenty-six structural variables were defined for the present study. Five arithmetic variables were selected for investigation from the Jerman (1972) study. They are defined as follows:

1. RECALL. The sum of the following:
   (a) One count was given for each formula to be recalled.
   (b) One count was given for each step in the formula.
   (c) One count was given for each conversion to be recalled and used.
   (d) One count was given for each fact from a previous problem to be recalled and used.

2. OPER2. The sum of the following:
   (a) One count was given for each different operation used.
   (b) Add four for one or more division operations.
   (c) Add two for one or more multiplication operations.
   (d) Add one for one or more audition operations.

3. OPER3. The sum of the following:
   (a) One count was given for each different operation used.
   (b) Add four for each division operation.
   (c) Add two for each multiplication operation.
   (d) Add one for each addition operation.

4. QUO. One of the following:
   (a) If division was used, one count was given for each digit of each quotient.
   (b) Zero, if division was not used.

5. NOMC2. One count was given for each regrouping that occurred in each multiplication.

Four well-defined independent variables were selected from Krushinski's study (1972). All linguistic variables are prefixed by the letters "LG" and they are defined as follows:

6. LGWORD. One count was given for each word in the problem statement. Numerals, e.g., 376.2, were given a count of one. Written expressions, e.g., thirty-two, were given a count of two.

7. LGSENT. One count was given for each sentence.

8. LGWDQU. One count was given for each word in the question sentence.

9. LGPREP. One count was given for each preposition in the problem statement.

Four new linguistic variables were defined for the present study as follows:

10. LGMXST. One count was given for each word in the longest sentence of the problem statement.

11. LGNUQU. One count was given for each numeral in the question sentence.

12. LGNMBR. One count was given for each numeral in the problem statement.

13. LGREL. One count was given for each numerical relationship stated in the problem.

Nine new algebraic variables were defined for the present study in terms of the equations expected to be used to solve each problem. Each of these nine algebraic variables was prefixed by the letters "EQ". Three different forms of each algebraic equation were used for determining the count for the "EQ" variables. The three forms are defined as follows:

Form (A). Unsimplified Equation:

Example: $\frac{3}{4}(X+2) + 6(2X) = 48$

Form (B). Simplified Equation: The parentheses were removed.

Example: $\frac{3X}{4} + \frac{6}{4} + 12X = 48$

Form (C). Canonical Equation: The equation was written in canonical form.

Example 1: $\frac{3X}{4} + \frac{6}{4} + 12X - 48 = 0$

Example 2: $0 = 48 - \frac{3X}{4} - \frac{6}{4} - 12X$

The "EQ" algebraic variables were defined as follows:

14. EQTRPZ. One count was given for each transposition required to isolate the terms that contained variables from the constant terms in the Form (B) equation. The count was taken before any combination of like terms.

15. EQPARA. One count was given for each set of parentheses in the Form (A) equation. Parenthetical expressions preceded by +1 or -1 were not counted.

16. EQXTRM. One count was given for each term that contained a variable in the Form (B) equation.

17. EQTOP. One count was given for each indicated arithmetic operation in the Form (A) equation.

18. EQCHAR. One count was given for each alphanumeric character in the Form (A) equation. One count was given for each decimal point, sign, and left or right parenthesis.

19. EQSIGN. The minimum of the two counts defined as follows:
    (a) One count was given for each positive term in the Form (C) equation.
    (b) One count was given for each negative term in the Form (C) equation.

20. EQPTRM. One count was given for each term within parentheses in the Form (A) equation. Parenthetical expressions preceded by a +1 or -1 were not counted.

21. EQDEC. One count was given for each decimal or fraction in the Form (B) equation.

22. EQANS. One count was given for each answer required in the problem statement.

Four additional algebraic variables were defined with respect to those factors that were involved in the writing of the equation; and often called the translation aspect of verbal problem solving. The four variables defined below were prefixed with the letters "TR".

23. TRTRAN. One count was given for each unknown that was used in the definition of another unknown.

24. TRTRMS. One count was given for each term of each unknown defined.

25. TRCPMT. One count was given if an unknown was defined as the complement of another unknown.

26. TRUKNS. One count was given for each unknown defined.

An example of the coding of variables for word problems is given below.

Example: The second angle of a triangle is twice the first angle of the triangle. The third angle is three times the second angle. Find the angles of the triangle.

A. Translations:
   Let X = the first angle,
   Then 2X = the second angle,
   and 3(2X) = the third angle.

B. Forms:

$X + 2X + 3(2X) = 180$	Form (A)
$X + 2X + 6X = 180$	Form (B)
$X + 2X + 6X - 180 = 0$	Form (C)

C. Solution:

$$X + 2X + 3(2X) = 180$$
$$X + 2X + 6X = 180$$
$$9X = 180$$
$$X = 180/9$$

Ans. $X = 20$

Ans. $2X = 40$

Ans. $3(2X) = 3(40) = 120$

D. Variable Coding:

Variable Number	1	2	3	4	5	6	7	8	9	10	11	12	13
Value	2	10	16	2	1	29	3	6	3	14	0	2	2

Variable Number	14	15	16	17	18	19	20	21	22	23	24	25	26
Value	0	1	3	5	13	1	1	0	3	2	3	0	3

Method

Ninety-six students enrolled in the Introduction to Algebra Course (MATH 4) at The Pennsylvania State University participated in the present study. The course was organized to permit each student to progress at an individual rate through each of 20 instructional units. The students took a computer-generated paper and pencil posttest at the end of each unit. The problems for each student's test were randomly selected from a file of problems prepared for each unit.

Unit tests 9, 13, and 14 were selected for the purposes of this study. Units 9, 13, and 14 contained word problems whose solutions involved first-degree equations in one unknown.

The 23 word problems selected for this study were those problems that at least five students attempted. The problem set consisted of one consecutive integer problem, two distance problems, three age problems, four angles of triangle problems, four direct variation problems, and seven miscellaneous problems.

Three direct variation problems had a percent correct of 100. The fourth direct variation problem had a percent correct of 80. Fourteen of the 26 structural variables had identical values for each of the four direct variation problems. The direct variation problems required evaluation of formula skills rather than the skills more common to the other problems selected, e.g. formulation of equations and solution of equations. Therefore, the four direct variation problems, the only four problems selected from unit 14, were eliminated from the problem set investigated in the present study.

A stepwise linear regression program, a modified version of BMDO2R (UCLA), was used to obtain regression coefficients, multiple correlations R and R^2. A clear explanation of the use of stepwise linear regression can be found in Spurr and Bonini (1967). A detailed explanation of the use of stepwise linear regression models with structural variables in problem solving research can be found in Suppes, Jerman, and Brian (1968) and Suppes, Loftus, and Jerman (1969).

Results

The mean percent correct for the 24 word problems was 60.74 and the standard deviation was 28.94.

The variables which entered the first 12 steps in the stepwise regression for the problem set are presented in Table 1. The multiple R, R^2, and increase in R^2 is given.

Insert Table 1 About Here

An approximate indication of the goodness of fit of the regression line was given by the multiple correlation coefficient, R, and R^2, which was an estimate of the amount of variance accounted for by the regression model. In the present study at step six about 80 percent of the variance was accounted for by the model; at step 12 about 94 percent of the variance was accounted for by the model.

Two translation variables, three linguistic variables, three arithmetic variables, and four equation variables entered on the first 12 steps in the stepwise linear regression.

The regression coefficient, standard error, computed t-value, and partial correlation coefficient for each of the first 12 variables to enter the stepwise regression are presented in Table 2.

Insert Table 2 About Here

The variable LGMXST, the number of words in the longest sentence, had the highest correlation with the observed probability correct of the word problem set. The variables EQPARA, LTTRAN, EQTOP, and LGREL were also

good predictors of the observed probability correct for the problem set. The variable RECALL entered the stepwise linear regression on the third step with a negative regression coefficient and a negative partial correlation coefficient.

The QUO variable's computed t-value was significant at the .001 level, but the QUO variable also had the lowest partial correlation coefficient. The variables LTTRAN, RECALL, EQTRPZ, and EQPARA were significant at the .01 level. Significant at the .05 level were the variables LGMXST, LTTRMS, EQXTRM, and EQTOP. The variables OPER2, LGREL, and LGNUOU were not significant at the .05 level.

Discussion

As the introduction indicated, the underlying purpose of the structural variable research has been to identify about six independent well-defined structural variables which permit a reasonably accurate prediction of the observed probability correct of word problems. Therefore, the present study was primarily interested in those structural variables which entered in the first six steps of the stepwise linear regression. The data for the six variables which entered in steps 7-12 of the stepwise regression were included for completeness. The discussion is restricted to the variables which entered in the first six steps of the regression. One linguistic variable (LGMXST), one translation variable (LTTRAN), one equation variable (EQTRPZ), and three arithmetic variables (RECALL, QUO, OPER2) comprised the first six entries.

The t-values of LGMXST, LTTRAN, RECALL, QUO, and EQTRPZ were significant at the .05 level or lower. The variable OPER2 was not significant at the .05 level.

The multiple R, R^2 values, and the significance of the computed t-values were quite encouraging. Other encouraging results were the entry of RECALL and QUO in the first six steps of the regression and the significance of their computed t-values. RECALL and QUO were robust variables in the research done using arithmetic word problems (Jerman, 1971; Jerman and Rees, 1972; Jerman and Mirman, 1972; Jerman, 1972).

The sign of the partial correlation coefficient of the RECALL variable was negative. This negative correlation might imply that those algebraic word problems involving the recall and use of a formula are easier. The recall of a formula might aid in the recognition of the problem type and the steps that should be taken to solve the algebra problem.

Jerman and Mirman (1972) and Jerman (1972) found the OPER3 variable, a weighted count on the number and type of operations necessary to solve a problem, entered in the first six steps of the stepwise linear regression; the OPER2 variable, a weighted count on the type of operations necessary to solve a problem, did not enter in one of the first six steps. In the present study the opposite situation occurred. The OPER2 variable entered in the first six steps but the OPER3 variable did not. Even when the OPER2 and OPER3 variables were analyzed independently the result was unchanged. The relative contribution of OPER2 versus OPER3 in the prediction of the observed probability correct in word problems is still unclear. Structural variables defined in terms of the different types of operations and the number of operations seem to have a definite robustness in the studies using arithmetic and algebra word problems. The affect of weighting

the types of operations versus weighting the type and number of operations should be investigated in future research.

The linguistic variable in the present study labelled LGWORD, the number of words in the problem statement, entered in one of the first six steps of the stepwise linear regression in several previous studies (Loftus, 1970; Jerman, 1971; Jerman and Mirman, 1972; Jerman and Rees, 1972; Jerman, 1972). The variable LGWORD did not enter in one of the first twelve steps of the regression in the present study. The linguistic variable LGMXST, the number of words in the longest sentence of the problem statement, entered in the first step of the stepwise regression with a significant t-value in the present study. The results of the present study, using LGMXST, and the previous studies cited, using LGWORD, indicate that algebraic and arithmetic word problems will be difficult to solve if they involve either lengthy sentences or lengthy problem statements, respectively. The relative contribution of lengthy sentences and problem statements to the variance of the observed probability correct of word problems should be systematically investigated in future studies.

The translation variable TRTRAN, the number of different unknowns that were used in the definition of other unknowns, entered in the second step and made significant contribution to the regression equation. This seems to imply that algebra word problems which involve a logical transitivity among the unknowns will be relatively more difficult to solve. On the basis of the present study a problem in which the third and second angles of a triangle are directly defined in terms of the first angle will be easier to solve than a problem in which the third angle is defined in terms of the second angle which is then defined in terms of the first angle.

The equation variable EQTRPZ, the number of transpositions required, entered in the fifth step of the regression and was significant. It appears that the number of transpositions is an important factor that varies directly with the difficulty of the algebraic word problems.

In terms of the present study the structural variables LGMXST. LTTRAN, RECALL, QUO, EQTRPZ, OPER2 were the most important in accounting for a maximum amount of the variance of the observed probability correct of algebraic word problems attempted by college students in an introductory algebra course.

The limitations of the study must be recognized. The subjects were college students, and some word problems were attempted by as few as five students. This study was an initial investigation to determine some structural variables that show promise for further refinement and research, and as such, the study fulfilled its role. A main objective for future studies is to increase considerably the number of students who attempt the problems. The writer would like to see some future investigations in secondary school algebra classes. Also, the variables should be tested using an entirely different set of algebra word problems. Research should be conducted to analyze the effect of systematically varying the definitions of some structural variables. If the structural variables prove to be robust, a set of problems should be generated that contain several predicted levels of difficulty and tested with students.

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TABLE 1

Order of Entry, R, R^2, Increase in R^2

for the 24 Problems

Step	Variable Name	Variable Number	R	R^2	Increase in R^2
1	LGMXST	10	0.5443	0.2962	0.2962
2	TRTRAN	23	0.7243	0.5247	0.2282
3	RECALL	1	0.7788	0.6066	0.0819
4	QUO	4	0.8187	0.6703	0.0637
5	EQTRPZ	14	0.8764	0.7682	0.0978
6	OPER2	2	0.8967	0.8040	0.0358
7	EQPARA	15	0.9067	0.8220	0.0180
8	LGREL	13	0.9235	0.8529	0.0309
9	TRTRMS	24	0.9341	0.8725	0.0196
10	EQXTRM	16	0.9471	0.8970	0.0246
11	EQTOP	17	0.9660	0.9331	0.0361
12	LGNUQU	11	0.9670	0.9350	0.0019

Table 2

Regression Coefficients, Standard Errors, of Regression Coefficients,

Computed T-Values, and Partial Correlation Coefficients

for the Problem Set

Step	Variable	Regression Coefficient	Standard Error	Computed T-Value	Partial Correlation Coefficient
1	LGMXST	0.08722	0.02862	3.047*	0.544
2	TRTRAN	2.30835	0.56626	4.076**	0.468
3	RECALL	-0.25610	0.07355	-3.482**	-0.290
4	QUO	0.56031	0.11833	4.735***	0.027
5	EQTRPZ	0.99023	0.27612	3.586**	0.293
6	OPER2	0.10931	0.07437	1.470	0.184
7	EQPARA	1.50927	0.34799	4.337**	0.487
8	LGREL	-0.35388	0.21778	-1.625	0.416
9	TRTRMS	-0.65485	0.24555	-2.667*	0.328
10	EQXTRM	1.15027	0.37336	3.081*	0.311
11	EQTOP	-0.32224	0.12742	-2.529*	0.436
12	LGNUQU	-0.05801	0.10201	-0.569	0.177
		C = -6.09106			

*p $<$ .05

**p $<$ .01

***p $<$ .001